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Equilibrium Competition, Social Welfare and Corruption in Procurement Auctions*

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Abstract

We study the effects of corruption on equilibrium competition and social welfare in a public procurement auction. In our model, firms are invited to the auction at positive costs, and a bureaucrat who runs the auction on behalf of a government may request a bribe from the winning firm. We first present the over-invitation results in the absence of corruption, in which more than a socially optimal number of firms will be invited. Second, we show that the effects of corruption on equilibrium outcomes vary across different forms of bribery. For a fixed bribe, corruption has no effect on equilibrium competition, although it does induce social welfare loss. For a proportional bribe, a corrupt bureaucrat may invite fewer or more firms to the auction depending on how much he weights his personal interest relative to the government payoff. Thus, corruption may result in either Pareto-improving or deteriorating allocations. Finally, we show that information disclosure may consistently induce more firms to be invited, regardless of whether there is corruption.

Key words: procurement auction; competition; corruption; fixed bribe; proportional bribe

JEL classification: D44, D73, H57

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1 Introduction

Public procurements account for a substantial part of economies worldwide. In the European Union (EU), for example, more than 250,000 public authorities spend approximately 14% of GDP on the purchase of services and supplies each year.[1] Corruption is a common concern in this context. To prevent corruption, most countries implement laws and regulations to guarantee necessary competition and transparency in public procurements. For instance, the EU requires a minimum of 52 days for a public contract notice in an open procedure, for which any business can submit a tender, and in a restricted procedure, for which only thoese who are pre-selected are invited to submit a tender, a public authority must invite at least five bidders to the competition process.[2] The belief underlying these rules is that competition may help improve efficiency and reduce corruption.

Surprisingly, the existing literature has paid insufficient attention to the intrinsic link between corruption and competition in public procurements. In this paper, we will investigate this important issue in a model of a procurement auction. Specifically, in our model, there are three parties: a government, such as the Department of Defense; a bureaucrat who runs the procurement auction on behalf of the government; and a number of potential firms (bidders) who can bid for the public contract if invited. Firms are invited to the auction at a positive invitation cost, and the bureaucrat may request a bribe from the winning firm of the auction.[3]

We investigate the common practice of first-price procurement auctions. In a standard first-price procurement auction, the firm with the lowest cost wins, and the price received is equal to its bid. Without corruption, the bureaucrat's objective is in line with that of the government, which is related to the price paid to the winning firm. The social welfare, however, is related to the actual production cost of the winning firm; thus, we do not model the government as a social planner in this paper.

Under the standard assumption for procurement auctions that firms' cost distribution is of decreasing reversed hazard rate (DRHR) and that there is a positive invitation cost, we show that in equilibrium, the bureaucrat will invite more firms than the socially optimum to bid for the public contract. In other words, the optimal number of firms that maximizes the government's payoff is larger than the efficient number of firms that maximizes social welfare. This over-invitation result is not as surprising as it first looks, as the government is modeled as a government division in our model; it cares about its own procurement payoff rather than the overall social welfare. The intuition is that

[1] EU website, http://ec.europa.eu/growth/single-market/public-procurement/index_en.htm.

[2] See europa.eu/youreurope/business/public-tenders/rules-procedures/index_en.htm. Updated on Nov. 2015.

[3] This practice apprears in all public sectors, and a portion of the sum that a winning contractor received is designated for the official as kickbacks. See the report prepared for the European Commission by PwC and Ecorys, *"Identifying and Reducing Corruption in Public Procurement in the EU."* June 30, 2013.

inviting an extra firm reduces the expected total surplus of the firms, which is ignored by the payoff-maximizing government but is taken into account in the total social welfare.

We then introduce corruption into the procurement auction, in which the bureaucrat may request a bribe from the winning firm. In public procurements, corruption takes many forms. In our study, we consider two specifications of bribery. The first is a fixed bribe in which the bureaucrat requests the winning firm to pay a fixed amount as a bribe. For instance, the fixed bribe can be in the form of a commission fee or a kickback that occurs in the real world (Inderst and Ottaviani, 2012). The second is a proportional bribe, whereby the winning firm must share a proportion of its revenue with the corrupt bureaucrat. For example, in Indonesia, the former president Suharto was publicly known as "Mr. Twenty-Five Percent" because he required that all major contracts throughout the nation give him 25 percent of the income.[4] We assume that the bureaucrat cares about a weighted average of his individual bribe and the government payoff.

Our main result is that the effects of corruption on equilibrium competition and social welfare vary across different forms of bribery. When the bribe is a fixed amount, the corrupt bureaucrat invites the same number of firms as in the absence of corruption, as his incentive to invite firms remains the same as before. That is, corruption in the form of a fixed bribe has no effect on equilibrium competition. However, it does change social welfare and resource allocation in equilibrium. Under the expectation of paying the fixed bribe upon winning, all firms will mark up their bids by the same amount of bribe, which increases the expected payment of the government to the winning firm. As a result, the fixed bribe is actually paid by the government, and it will not hurt the firms at all. Meanwhile, increased public expenditure by the government implies social welfare loss due to the marginal cost of distortion of public funds.

By contrast, under a proportional bribe, the corrupt bureaucrat may invite either fewer or more firms to the auction than before, depending on how much the bureaucrat weights his personal interest relative to the government payoff. There are two opposite effects here on the equilibrium competition. On the one hand, the proportional bribe will increase firms' bids proportionally, therefore making the distribution of bids more dispersed. It decreases competition in the auction, and, in response to it, the bureaucrat needs to encourage competition. On the other hand, the bureaucrat also has an incentive to discourage competition, as the winning firm's revenue is decreasing in competition in the auction, and the bribe he receives is a proportion of it. Therefore, the relative magnitude between these two opposite effects determines the equilibrium level of competition. A seemly surprising result is that corruption in the form of a proportional bribe may induce Pareto-improving allocation in equilibrium.

We also consider the format of second-price procurement auctions and show that

[4] Wrage, Alexandra Addison. *Bribery and Extortion: Undermining Business, Governments, and Security.* Westport, Conn.: Praeger Security International, 2007. p. 14.

the Revenue Equivalence Theorem still holds in our settings. Specifically, under the fixed bribe, firms will mark up their bids by the same amount of bribe, and under the proportional bribe, firms will correspondingly raise their bids proportionally. In the end, the effects of corruption on equilibrium competition and social welfare are the same in both first- and second-price procurement auctions.

We further investigate the effects of information disclosure on auction outcomes. When firms' areas of specialization are differentiated, revealing project information may induce more dispersed distribution of firms' cost estimates. A piece of project information may drive up the cost estimates of some firms, if they find it is a mismatch to their areas of specialization, while driving down those of others that find it to be a good match. As a result, firms' cost estimates become more dispersed under information disclosure. We show that information disclosure increases both the efficient and optimal number of firms in procurement auctions. The intuition is that under information disclosure, firms' cost estimates become more dispersed, and the auction becomes less competitive than before. It is then better to invite more firms to the auction, either to maximize government payoff or social welfare. This result continues to hold in the case of corruption.

Finally, we also provide brief discussions on the policy implications of our results. For the regulation of public procurement, we show that imposing a requirement on the minimum number of bidders may be effective only when it lies in a reasonable range. For instance, if it is too low, it will not impose real restrictions on a corrupt bureaucrat's choice; if it is too high, it may instead incur social welfare loss in equilibrium.

To the best of our knowledge, our study is the first to examine the effects of various forms of corruption on equilibrium outcomes in procurement auctions with a variable number of firms. In the literature, most papers focus on how a bureaucrat manipulates the auction rules in exchange for bribes while the number of firms is fixed. For example, Compte et al. (2005) and Menezes and Monteiro (2006) consider a corruption model in which the bureaucrat may offer a favored firm an opportunity to readjust its initial bid in exchange for a bribe. This arrangement is known as *right of first refusal*, and other related papers include Burguet and Perry (2009) and Arozamena and Weinschelbaum (2009). Another strand in the literature studies corruption in multidimensional procurement auctions, whereby the government may care about both the price and quality of the project. Celentani and Ganuza (2002) and Burguet and Che (2004) examine corruption in which the bureaucrat manipulates the quality assessment to favor the firms offering higher bribes.

Regarding the link between competition and corruption, the general idea is that increasing competition may reduce corruption (Svensson, 2005), and Ades and Di Tella (1999) provide some supportive empirical evidence showing that corruption is higher in countries where foreign competition is restricted. However, several theoretical studies show that a simple relationship may not hold generally. Bliss and Di Tella (1997) propose a model in which both the equilibrium number of firms and the level of corruption are endogenously determined by other parameters, and the negative relationship between competition and corruption does not always hold. Celentani and Ganuza (2002) also find that with an increasing number of firms, there may be a higher level of corruption. In this paper, we also show that the relationship between corruption and competition depends on the particular form of bribery.

Our paper is also related to the literature on auctions with costly entry. McAfee and McMillan (1987) and Levin and Smith (1994) examine the entry process in which potential bidders must pay a fixed entry cost. Szech (2011) and Fang and Li (2015) consider the other case in which the auctioneer incurs costs to invite potential bidders, and they show the over-invitation result in ascending auctions when bidders' valuation distribution is of increasing failure rate. We follow the similar setting of positive invitation cost, albeit in the case of descending procurement auction, and derive the over-invitation results under the different DRHR assumption. Our primary focus in on the link between corruption and the equilibrium competition and social welfare in procurement auctions.

The remainder of this paper is organized as follows. Section 2 introduces the benchmark model of procurement auction where there is no corruption. Section 3 studies the equilibrium outcomes under corruption, and there are two cases, the fixed bribe and proportional bribe. Section 4 provides some further discussions, and Section 5 presents our concluding remarks.

2 Benchmark: without corruption

Consider a public procurement auction in which a government plans to allocate a contract, and a bureaucrat runs the auction on behalf of the government. We assume there are an infinite number of potential firms qualified for this contract, and n firms are invited to the auction at a cost of $C(n)$, which is paid by the government. The value of the contract to the government is V. The cost function $C(n)$ is increasing and weakly convex, that is, $C'(n) > 0$ and $C''(n) \geq 0$. The firms are *ex ante* homogenous whose production cost X conforms to the distribution of $F(\cdot)$ on $[0, V]$, with strictly positive density $f(\cdot)$. Assume all players are risk-neutral.

For those invited firms, $i = 1, 2, ..., n$, let $\{X_i\}_{i=1}^n$ be n independent draws from the same distribution $F(\cdot)$, where X_i is the production cost for firm i. The distribution of $F(\cdot)$ is common knowledge, and the realization of X_i is only observed by firm i. We denote $X_{k:n}$ be the kth lowest cost of the n invited firms, and we have

$$X_{1:n} \leq X_{2:n} \leq \cdots \leq X_{n:n}.$$

For an order statistic $X_{k:n}$, let $F_{k:n}(\cdot)$ and $f_{k:n}(\cdot)$ be its cumulative distribution function and probability density function, respectively.

As commonly practiced, we assume that the procurement runs in the format of a sealed-bid *first-price auction*. It is well known that the symmetric equilibrium bidding strategy in a first-price procurement auction is given by (e.g., Krishna, 2002, p.17),

$$b(x) = E[X_{1:n-1}|X_{1:n-1} > x] = x + \int_x^V \frac{\bar{F}_{1:n-1}(y)}{\bar{F}_{1:n-1}(x)} dy,$$

where $\bar{F}_{1:n-1}(x) = 1 - F_{1:n-1}(x)$ is the survival function.

In the auction, the firm with the lowest cost $X_{1:n}$ wins, and its revenue is equal to the bid $b(X_{1:n})$.[5] The rent for the winning firm is thus

$$R(n) = b(X_{1:n}) - X_{1:n}. \tag{1}$$

The government's payoff is equal to the net project benefit, $V - b(X_{1:n})$, minus the invitation cost, $C(n)$, which is

$$\Pi(n) = V - b(X_{1:n}) - C(n). \tag{2}$$

In this benchmark case of no corruption, we assume the bureaucrat's objective is in line with that of the government, and he does not gain personal benefit.

It is worth noting that we do not model the government as a social planner. Instead, we model it as a government agency that cares about its own payoff rather than overall social welfare. For example, the Department of Defense in the U.S. procures weapons systems, and its primary concerns are the budget spending and the performance of the weapons system procured.

Government spending, including payment to the winning firm, $b(X_{1:n})$, and the expenditure on invitation cost, $C(n)$, uses public funds that are financed through taxation. We assume that there is a marginal cost of public funds, which is $\lambda \in [0, 1)$, due to the distortion of resource allocation caused by taxation (Laffont and Tirole, 1987). Social welfare is then measured by the sum of the rents of firms (1) and the payoff of the government (2), less the distortion cost of public funds, which is

$$W(n) = R(n) + \Pi(n) - \lambda(C(n) + b(X_{1:n})) = V - X_{1:n} - \lambda b(X_{1:n}) - (1 + \lambda)C(n). \tag{3}$$

As the auction organizer, the bureaucrat decides how many firms are invited. When there is no corruption, his objective is to select the optimal number of firms to maximize the expected government payoff, $E[\Pi(n)]$. We are also interested in the efficient number of firms that maximizes the expected social welfare, $E[W(n)]$. Next, we will show that the

[5] The case of tying is ignored since the distribution of firms' production costs is continuous.

optimization problems of $E[\Pi(n)]$ and $E[W(n)]$ are well defined under specific standard assumptions, and these results are determined by the properties of $E[X_{1:n}]$ and $E[X_{2:n}]$. We provide several results in the following paragraphs.

Lemma 1 *The expected payment of the government is equal to the expected second-lowest production cost, that is,*

$$E[b(X_{1:n})] = E[X_{2:n}].$$

This result is implied by the Revenue Equivalence Theorem.

Lemma 2 $E[X_{1:n}]$ *is strictly decreasing and strictly convex in* n*, and* $\lim_{n\to\infty} E[X_{1:n}] = 0$*.*

Proof: In the Appendix. ■

The monotonicity property is straightforward as $X_{1:n}$ is the smallest order statistic, and as the number of firms invited into the auction increases, $E[X_{1:n}]$ will naturally decrease and converge to zero. The convexity result is implied by the continuity of the distribution $F(\cdot)$. The property of the second-order statistic, $E[X_{2:n}]$, is more challenging to interpret. We will next show that as the distribution of X satisfies the property of decreasing reversed hazard rate, then $E[X_{2:n}]$ is strictly convex in n.

Definition 1 *The distribution of* X *is said to be of decreasing reversed hazard rate (DRHR) if its reversed hazard rate*

$$\frac{f(x)}{F(x)}$$

is decreasing in x*.*

The DRHR assumption on X in the procurement auction is analogous to the regularity condition of increasing failure rate in the standard ascending price auction (Myerson, 1981). The difference is that in a procurement auction, the bidder offering the lowest bid wins the auction, whereas in a standard ascending price auction the bidder offering the highest bid wins.

There are many examples of DRHR distributions, such as uniform, normal and exponential distributions. Furthermore, a positive random variable cannot have increasing reversed hazard rate, because $f(x)/F(x)$ will converge to infinity when x approaches zero. Therefore, in our case of positive production cost, it is not possible for X to be of increasing reversed hazard rate.

The following result shows that when the distribution of X is of DRHR, $E[X_{2:n}]$ is strictly convex in n.

Lemma 3 *If the distribution of X is of DRHR, then*

(i) $E[X_{2:n}]$ *is strictly decreasing and strictly convex in n.*

(ii) *Moreover,* $\lim_{n\to\infty} E[X_{2:n} - X_{2:n+1}] = 0$.

Proof: In the Appendix. ■

The result that $E[X_{2:n}]$ is strictly decreasing in n does not depend on the condition of DRHR. In fact, DRHR is a sufficient yet unnecessary condition for the convexity result, and a weaker sufficient condition is that $\frac{F^2(x)}{f(x)}$ is increasing in x, which is implied by DRHR.

In addition, we are also interested in how the expected rent of the winning firm changes with n. From Lemma 1, it is the expectation difference of the first- and second-order statistic. We have the following result.

Lemma 4 *If the distribution of X is of DRHR, then the expected rent of the winning firm,*

$$E[R(n)] = E[X_{2:n} - X_{1:n}]$$

(i) *is strictly decreasing and strictly convex in n.*

(ii) $\lim_{n\to\infty} E[R(n)] = 0$.

Proof: In the Appendix. ■

The intuition underlying this result is that increasing competition will gradually squeeze out the expected rent of the winning firm. Furthermore, the expected rent converges to zero when the number of firms approaches infinity. A direct implication of Lemma 4 is that

$$E[X_{2:n} - X_{2:n+1}] > E[X_{1:n} - X_{1:n+1}], \tag{4}$$

that is, for given n, if one more firm is invited into the auction, then the expected value of $E[X_{2:n}]$ drops faster than $E[X_{1:n}]$.

Our Lemma 2-4 are analogous to Lemma 1-4 in Szech (2011), who studied second-price ascending auctions. In this paper, however, we examine the procurement auction in the format of first-price descending auctions. Based on our discussions above, we know that $\max E[\Pi(n)]$ and $\max E[W(n)]$ are both well defined concave maximization problems under the assumption of DRHR. Denote n^* to be the optimal number of firms that maximizes the government's expected payoff, and n^{**} to be the efficient number of firms that maximizes the expected social welfare, respectively. By definition, we have

$$n^* = \arg\max_n E[\Pi(n)] \text{ and } n^{**} = \arg\max_n E[W(n)]. \tag{5}$$

Here, the number of firms is discrete, and n^* and n^{**} may not be unique.

The main result of this section is the comparison between the optimal and efficient number of firms in the procurement auction without corruption, which gives us the following result.

Proposition 1 *If the distribution of X is of DRHR, then we have*

$$n^* \geq n^{**}. \tag{6}$$

Proof: In the Appendix. ■

The proposition states that when the distribution of firms' cost estimates is of DRHR, the bureaucrat, with the aim of maximizing the expected payoff for the government, will invite more firms into the procurement auction than the social optimum. In other words, over-invitation of firms occurs in the benchmark case of no corruption.

The over-invitation result is not surprising at first glance, and it is implied by the inequality function of (4). For one more firm entering the auction, the marginal change in the expected government payoff is $\Delta E[\Pi(n)] = E[X_{2:n} - X_{2:n+1}] - (C(n+1) - C(n))$, and the marginal change of the expected social welfare can be normalized as

$$\frac{1}{1+\lambda}\Delta E[W(n)] = \frac{1}{1+\lambda}E[X_{1:n} - X_{1:n+1}] + \frac{\lambda}{1+\lambda}E[X_{2:n} - X_{2:n+1}] - (C(n+1) - C(n)).$$

The inequality function (4) implies that for any given n, $\Delta E[\Pi(n)]$ is larger than $\frac{1}{1+\lambda}\Delta E[W(n)]$. Applying the optimization conditions of $\Delta E[\Pi(n^*)] \geq 0 > \Delta E[\Pi(n^*+1)]$ and $\frac{1}{1+\lambda}\Delta E[W(n^{**})] \geq 0 > \frac{1}{1+\lambda}\Delta E[W(n^{**}+1)]$, we obtain the over-invitation result (6). Basically, it is based on the fact that the marginal change of the expected government payoff is larger than the normalized marginal change of social welfare. Thus, the expected government payoff approaches the maximum more slowly than the expected social welfare does.

To gain a deeper understanding of the results obtained above, we provide a simple numerical example where X conforms to a uniform distribution.

Example 1 Let $X \sim U[0, V]$, which is obviously of DRHR. If there are n firms in the auction, then

$$E[X_{1:n}] = V - \int_0^V F_{1:n}(x)dx = \frac{1}{n+1}V,$$

$$E[X_{2:n}] = V - \int_0^V F_{2:n}(x)dx = \frac{2}{n+1}V.$$

And both $E[X_{1:n}]$ and $E[X_{2:n}]$ are strictly decreasing and convex in n, as shown in Lemma 2 and 3. Assume $C(n) = nc$, where c is constant.

The expected rent of the winning firm is

$$E[R(n)] = E[X_{2:n} - X_{1:n}] = \frac{1}{n+1}V,$$

which is strictly decreasing and convex in n, and $\lim_{n\to\infty} E[R(n)] = 0$, as shown in Lemma 4.

The optimal decision problem for the bureaucrat is thus

$$\max_n E[\Pi(n)] = V - E[X_{2:n}] - nc = V - \frac{2}{n+1}V - nc,$$

and the optimal condition for n^* is

$$n^*(n^*+1) \le \frac{2V}{c} < (n^*+1)(n^*+2).$$

We next solve the following problem for the socially efficient number of firms:

$$\max_n E[W(n)] = V - E[X_{1:n}] - \lambda E[X_{2:n}] - (1+\lambda)nc.$$

The optimal condition for n^{**} is thus

$$n^{**}(n^{**}+1) \le \frac{(1+2\lambda)V}{(1+\lambda)c} < (n^{**}+1)(n^{**}+2).$$

Apparently, $n^* \ge n^{**}$, as $\frac{1+2\lambda}{1+\lambda} \le 2$. For example, if $\lambda = 0.2$, $c = 2$ and $V = 36$, then we have $n^* = 5$ and $n^{**} = 4$. This result confirms the result (6) in Proposition 1.

3 Corruption in Procurement Auctions

We next introduce corruption into the public procurement auction. As Jain (2001) states, corruption is an act in which the power of public office is used for personal gain in a manner that contravenes the rules of the game. Various types of corruption are identified in the real world. In this paper, we consider the case whereby the bureaucrat in charge of the procurement can request a bribe of $B(n)$ from the winning firm. We assume that the invited firms are fully aware of the request and that they accept this tacit rule prior to the auction. As only the winner pays the bribe, our setting rules out sunk investments for the firms as lobbying activities, as in all pay auctions or other standard rent-seeking models.

In particular, we consider two forms of bribery: a fixed bribe amount, $B(n) = B$, and a proportional bribe, $B(n) = \eta b(X_{1:n})$, $\eta \in [0, 1]$. In the first case, the fixed bribe amount is specified by the bureaucrat, and all the firms know this condition before offering their bids. In the second case, the fraction η is exogenously given, and the kickback is just a proportion of the winning bid. These two forms of bribery are relatively pervasive in the real practice of public procurements, such as commissions and kickbacks paid to agencies.[6]

We assume that the bureaucrat cares about not only his individual bribe $B(n)$ but also the government payoff, $\Pi(n)$. Specifically, the bureaucrat's objective function is a weighted average of these two terms, as follows:

$$U(n) = \alpha\Pi(n) + (1-\alpha)B(n),$$

where $\alpha \in [0, 1]$ is the weight for government payoff.

We do not consider a penalty for corruption in our model, although it could be introduced in a straightforward way, for example, a detecting probability that is decreasing in α and a penalty that is proportional to $B(n)$. Our focus here is on the relationship between corruption and equilibrium competition in the procurement auction. The introduction of a penalty will at most temper but will not change the direction of the results; therefore, we avoid that complication in our model.

3.1 Fixed bribe

In the case of fixed bribe, the bureaucrat requests the winning firm to pay a fixed bribe of B after the auction. As mentioned above, we assume that firms are fully aware of the request prior to the auction, and accept it as a tacit rule. The fixed amount of bribe B therefore appears to be a part of the cost, conditional on winning the auction. Let subscript 'F' denote fixed bribe. The virtual cost for firm i is now $X_i' = X_i + B$, conditional on winning. Thus, the symmetric equilibrium bidding strategy is $b_F(X_i) = b(X_i')$, which satisfies the following property:

Lemma 5 *The symmetric equilibrium bidding strategy in a first-price procurement auction, with a fixed bribe of B, is given by*

$$b_F(x) = E[X_{1:n-1}|X_{1:n-1} > x] + B = b(x) + B.$$

That is, all firms will increase the bid by the amount of the bribe.

Proof: In the Appendix. ■

As a result, the lowest cost firm wins with a bid of $b(X_{1:n}) + B$, and the payment it receives is the same as the bid. Thus, the rent for the winning firm remains the same as

[6] For example, in the case of Mexico City - Querétaro High-Speed Railway, it was said that the winning consortium gave a USD 7 million house, as a present to the president's wife. (See *Forbes*, Feb. 10, 2015. "Mexico Suspends Multibillion Dollar High-Speed Rail Project At Center of Political Scandal.") In the case of "Fat Leonard" procurement corruption scandal in US Navy, Leonard Francis of Singapore-based Glenn Defense Marine Asia admitted bribing US Navy officers tens of millions of dollars to win hundreds of millions in business and over-payments. (See *The New York Times*, Nov. 29, 2013. "Scandal Widens Over Contracts for Navy Work.")

in equation (1), which is

$$R_F(n) = b(X_{1:n}) - X_{1:n}. \tag{7}$$

However, under the fixed bribe, the government needs to pay the amount of B more than before in equation (2), and its payoff is now

$$\Pi_F(n) = V - B - b(X_{1:n}) - C(n). \tag{8}$$

The social welfare is equal to the sum of the rent of the winning firm (7), the government's payoff (8), and the fixed bribe for the bureaucrat B, and then minus the distortion cost of public funds $\lambda(B + b(X_{1:n}) + C(n))$. Thus, the total social welfare is

$$W_F(n) = V - X_{1:n} - \lambda(B + b(X_{1:n})) - (1+\lambda)C(n). \tag{9}$$

Compared with equation (3), the social welfare in the case of no corruption, we find that $W_F(n)$ is smaller than $W(n)$ by the extra cost of λB.

One potential issue is that the payoff for the government might be negative. If we think of V as the government's willingness to pay, then V can also be interpreted as the maximum budget for the contract. The case of negative government's payoff then can be explained as a case of a project budget deficit, which is relatively pervasive in public procurement. See Ganuza (2007) for more discussions on the cost overruns in procurements. Here, we allow the possibility of negative $\Pi_F(n)$ in our model.[7]

The corrupt bureaucrat's optimization problem is

$$\max E[U_F(n)] = \alpha E[\Pi_F(n)] + (1-\alpha)B. \tag{10}$$

Apparently, when the distribution of X is of DRHR, $E[\Pi_F(n)]$ is strictly increasing and concave in n. The optimization conditions for equation (10) are thus

$$E[X_{2:n-1} - X_{2:n}] \geq C(n) - C(n-1)$$

and

$$E[X_{2:n} - X_{2:n+1}] < C(n+1) - C(n),$$

which are equivalent to the optimization conditions in the case of no corruption. If we denote the optimal number of firms in this case of fixed bribe as n_F^*, then the following result is obvious.

[7] Analogous to the participation condition, we can assume the expected rent of the winning firm and the expected payoff to the government should be non-negative. It is reasonable to say that only $E[\Pi(n)] \geq 0$ and $E[\Pi_F(n)] \geq 0$ needed to be considered. In particular, as the bureaucrat makes choices on the number of invited firms and the fixed bribe amount, these restrictions are applied. That is, the bribe amount is bounded by the expected government payoff.

Proposition 2 *If the distribution of X is of DRHR, then under a fixed bribe, a corrupt bureaucrat will invite the same number of firms into the procurement auction, as in the case of no corruption. That is,*

$$n_F^* = n^*. \tag{11}$$

However, the fixed bribe incurs a social welfare loss of λB.

The result indicates that bribery in the form of a fixed bribe has no effect on equilibrium competition. The intuition underlying this result is straightforward. As the bribe is fixed at a given level, the bureaucrat's objective remains the same as in the benchmark case of no corruption, which is to maximize the expected government payoff. Therefore, the fixed bribe will not change the incentive for the corrupt bureaucrat to invite the firms. However, it changes resource allocation and the social welfare in equilibrium. As all the firms mark up their bid by the fixed amount of bribe B, the bribe is actually paid by the government and will not hurt the firms. Furthermore, as the government expenditure increases by B in this case, the distortion cost of public funds implies that the amount of social welfare loss is λB. In other words, we then have $W_F(n_F^*) - W(n^*) = -\lambda B$ from equations (3), (9) and (11).

In this case, of a fixed bribe, as mentioned before, the expected government payoff could be negative. We interpret this case as a project budget deficit for the government. Alternatively, if we impose the *ex ante* participation constraint for the government, the bureaucrat's decision is subject to the constraint that $E[\Pi_F(n)] \geq 0$. In this case, the value of the fixed bribe B is determined endogenously. However, the result will remain the same as in Proposition 2. For instance, let $B(n) = E[\Pi(n)]$, it implies $E[U_F(n)] = (1-\alpha)E[\Pi(n)]$, and thus, we also have $n_F^* = n^*$.

3.2 Proportional bribe

We next consider the case of proportional bribe, and assume that the winning firm needs to pay a proportion of $\eta \in [0,1]$ of its winning bid to the corrupt bureaucrat. The intuition implies that firms will exaggerate their bids, and the following result shows that the ratio of exaggeration is $\frac{1}{1-\eta}$. Let us first derive the symmetric equilibrium bidding strategies for firms.

Lemma 6 *The symmetric equilibrium bidding strategy in a first-price procurement auction, with a proportion bribe, is given by*

$$b_P(x) = \frac{1}{1-\eta} E[X_{1:n-1} | X_{1:n-1} > x] = \frac{b(x)}{1-\eta}.$$

That is, all firms will increase the bid by the ratio of $\frac{1}{1-\eta}$.

Proof: In the Appendix. ■

Under the proportional bribe, the rent for the winning firm is

$$R_P(n) = (1-\eta)b_P(X_{1:n}) - X_{1:n} = b(X_{1:n}) - X_{1:n},$$

where the subscript 'P' denotes the case of proportional bribe. It is obvious that the government's payoff function is no longer the same as in equation (2), and it is now

$$\Pi_P(n) = V - b_P(X_{1:n}) - C(n) = V - \frac{b(X_{1:n})}{1-\eta} - C(n).$$

The corrupt bureaucrat receives the bribe $B(n) = \eta b_P(X_{1:n})$. As before, the total social welfare is the sum of the payoffs of the three parties, minus the distortion cost of public funds, which is

$$W_P(n) = V - X_{1:n} - \frac{\lambda}{1-\eta} b(X_{1:n}) - (1+\lambda)C(n).$$

The corrupt bureaucrat's payoff function is a weighted average of his bribe and the government's payoff, and his problem is thus

$$\max E[U_P(n)] = \alpha E[\Pi_P(n)] + (1-\alpha)\eta E[b_P(X_{1:n})]. \tag{12}$$

Note that the concavity of $E[U_P(n)]$ is not guaranteed naturally in this case. Under the DRHR assumption, we know that $E[\Pi_P(n)]$ is concave, and $\eta E[b_P(X_{1:n})]$ is convex in n, and therefore the convex combination of them is not necessarily concave. Simple transformation follows another expression of $E[U_P(n)]$ as

$$E[U_P(n)] = \alpha(V - C(n)) - \frac{\alpha - (1-\alpha)\eta}{1-\eta} E[X_{2:n}].$$

From Lemma 3, we know $E[X_{2:n}]$ is strictly decreasing and convex in n, and thus $E[U_P(n)]$ is strictly concave if $\alpha - (1-\alpha)\eta > 0$.

Lemma 7 *If the distribution of X is of DRHR and $\eta < \frac{\alpha}{1-\alpha}$, then $E[U_P(n)]$ is strictly concave in n.*

In practice, the condition of $\eta < \frac{\alpha}{1-\alpha}$ should readily be satisfied in the case of public procurement. Given the magnitude of public procurement, usually tens of millions USD, we could reasonably think that the bribe amount is small relative to the winning bid. In fact, it is trivial if $\eta \geq \frac{\alpha}{1-\alpha}$, the bureaucrat's objective function, $E[U_P(n)]$, is strictly decreasing in n, and then the optimal number of firms, denoted by n_P^*, is equal to 1, i.e., the *single bid* scenario. Here, we focus on the condition of $0 < \eta < \frac{\alpha}{1-\alpha}$, and the following result holds.

Proposition 3 *If the distribution of X is of DRHR and $0 < \eta < \frac{\alpha}{1-\alpha}$, then under a proportional bribe, a corrupt bureaucrat invites either fewer or more firms into the procurement auction than in the benchmark case of no corruption. Specifically,*
(i) *if $\alpha \in [0, 1/2]$, then $n_P^* \leq n^*$;*
(ii) *if $\alpha \in [1/2, 1]$, then $n_P^* \geq n^*$.*

Proof: In the Appendix. ■

The optimal number of firms under a proportional bribe can be either larger or smaller than the competition without corruption, which depends on the relative magnitude of two opposite effects: i) the first term of equation (12), $E[\Pi_P(n)]$, implies that he may encourage competition, as firms raise their bids and therefore the distribution of bids turns out to be more dispersed than before, which decreases the level of competition in the auction; ii) the second term of equation (12) implies that the bureaucrat may discourage competition, as $E[b_P(X_{1:n})]$ is decreasing in n. Thus, the effect of the proportional bribe on the competition depends on how much the bureaucrat weights his individual interest relative to the government payoff. As a result, when the corrupt bureaucrat cares more about his personal interest, $\alpha < 1/2$, he will dampen competition in the auction, and when he cares more about the government payoff, $\alpha > 1/2$, he will encourage competition.

Finally, let us turn to the socially efficient number of firms in different cases. If we denote the socially optimal number of firms in the case of fixed and proportional bribe by n_F^{**} and n_P^{**} respectively, then we have following result.

Proposition 4 *The order of socially efficient numbers of firms is*

$$n^{**} = n_F^{**} < n_P^{**}.$$

Proof: In the Appendix. ■

This result suggests that the effects of bribery on efficient competition vary on the formats of bribery. Although the fixed bribe does not change the socially efficient competition, the bribery incurs some social cost. Specifically, we have

$$W_F(n_F^{**}) + \lambda B = W(n^{**}),$$

from equations (3) and (9). Under proportional bribery, firms exaggerate their bids and the distribution of the winning bids becomes more dispersed, which implies a larger number of firms for social efficiency.

Let us extend the previous numerical example to the case of corruption, which helps us to understand the various effects of corruption.

Example 2 As we assume $X \sim U[0, V]$, we have already known that

$$E[X_{1:n}] = \frac{1}{n+1}V \text{ and } E[X_{2:n}] = \frac{2}{n+1}V.$$

In the case of a fixed bribe, we have

$$E[U_F(n)] = \alpha E[\Pi_F(n)] + (1-\alpha)B.$$

The optimal number of firms is apparently $n_F^* = n^*$ as shown in Proposition 2.

In the case of a proportional bribe, we have

$$E[U_P(n)] = \alpha(V - C(n)) - \frac{\alpha - (1-\alpha)\eta}{1-\eta}E[X_{2:n}] = [\alpha - \frac{\alpha-(1-\alpha)\eta}{1-\eta}\frac{2}{n+1}]V - \alpha nc.$$

If $\eta \geq \frac{\alpha}{1-\alpha}$, then $E[U_P(n)]$ is strictly decreasing in n, so $n_P^* = 1$, that is the minimum possible number of firms. It is clear that $\alpha = 0$ is a special case for the single bid result.

Let us consider $0 < \eta < \frac{\alpha}{1-\alpha}$, then $E[U_P(n)]$ is concave in n. The optimal condition for n_P^* is

$$n(n+1) \leq \frac{\alpha - (1-\alpha)\eta}{(1-\eta)\alpha} \cdot \frac{2V}{c} < (n+1)(n+2).$$

First, if $\alpha \leq 1/2$, we have $\frac{\alpha-(1-\alpha)\eta}{(1-\eta)\alpha} \leq 1$, and then $n_P^* \leq n^*$. For example, if $c = 2$ and $V = 36$, we have already known $n^* = 5$. Let us now select $\alpha = 1/3$ and $\eta = 1/4$ such that $\frac{\alpha-(1-\alpha)\eta}{(1-\eta)\alpha} = 2/3$, we have the optimal number of firms under the proportional bribe, $n_P^* = 4$, which is less than $n^* = 5$.

Second, if $\alpha \geq 1/2$, we have $\frac{\alpha-(1-\alpha)\eta}{(1-\eta)\alpha} \geq 1$, and then $n_P^* \geq n^*$. For example, if $c = 2$ and $V = 36$, and then $n^* = 5$. Now we select $\alpha = 2/3$ and $\eta = 2/5$ such that $\frac{\alpha-(1-\alpha)\eta}{(1-\eta)\alpha} = 4/3$, we have the optimal number of firms under the proportional bribe, $n_P^* = 6$, which is greater than 5.

4 Further discussion

4.1 Second-price procurement auctions

It is a natural question whether corruption has different effects on competition in the second-price procurement auction. As we know, bidding the true production cost X_i is a weakly dominant strategy in the second-price auction. In the benchmark case of no corruption, it is easy to show that the result of Proposition 1 holds in the second-price scenario. We now consider the results as corruption exists.

In the case of fixed bribe, the bureaucrat requests the winning firm to pay a fixed bribe of B after the auction, and then the fixed bribe amount appears to be a part of firms' cost condinional on winning the auction. As the virtual cost for firm i is now $X_i' = X_i + B$, bidding X_i' is thus a weakly dominant strategy for firm i, and all firms will mark up their bids by the fixed amount of B. Therefore, the fixed bribe has the same effect on the bidding function from Lemma 5, and we can derive the same result of Proposition 2 in the second-price setting.

Considering the case of a proportional bribe, we assume that the winning firm needs to pay a proportion of η of its revenue to the corrupt bureaucrat. We have the following result:

Lemma 8 *If the winning firm needs to pay a proportion of η of its revenue to the corrupt bureaucrat in the second-price procurement auction, bidding $\frac{1}{1-\eta}$ times the production cost is a weakly dominant strategy.*

Proof: In the Appendix. ■

Recalling Lemma 6, it is obvious that the results of a proportional bribe all remain the same in the second-price setting. In sum, the Revenue Equivalence result between the first- and second-price auctions still holds as corruption exists.

4.2 Information disclosure

As shown in the recent literature (e.g., Johnson and Myatt, 2006; Ganuza and Panelva, 2010; Jewitt and Li, 2015), information disclosure may induce more dispersed distribution of consumers' valuations.[8] The intuition is that when consumers' preferences are differentiated, revealing product information may drive up the valuations of some consumers, while driving down those of others; therefore, the distribution of posterior valuations becomes more dispersed.

A similar story can be applied here in our procurement context. For example, the firms may have different expertise or areas of specialization, and revealing information on the details of the public contract may have differentiated impacts on their cost estimates. That information may be good news for some firms when they find it is a good match for their areas of specialization, whereas it may be bad news for others. Consequently, the distribution of firms' cost estimates becomes more dispersed under information disclosure. Let us denote Y as the new cost for the firms after information disclosure, and the corresponding distribution is $G(\cdot)$. Compared with the initial cost of X with distribution $F(\cdot)$, we know the distribution of Y is more dispersed, or formally, $X \preceq_{disp} Y$, defined as follows:

[8] If information disclosure leads to more concentrated cost estimates, the opposite results are followed.

Definition 2 *The random variable X is smaller than Y in the dispersive order, denoted by $X \preceq_{disp} Y$, if*

$$F^{-1}(q) - F^{-1}(p) \leq G^{-1}(q) - G^{-1}(p)$$

for all $0 < p < q < 1$.

In Proposition 2 of Szech (2011), both the revenue-maximizer and the welfare-maximizer in advertising auctions attract more bidders for the more dispersed distribution of the bidders' valuations. The similar results hold in our settings. Moreover, we show that these results also hold when corruption exists. In summary, for the purpose of either welfare or payoff maximization, when the distribution of firms' costs become more dispersed under information disclosure, more firms need to be invited for the competition in the public procurement, whether it is with corruption or not, or whether the corruption is in the form of fixed bribe or proportional bribe.

Proposition 5 *If $X \preceq_{disp} Y$, and the distribution of $F(\cdot)$ and $G(\cdot)$ are DRHR, then we have*

$$n^* \leq \hat{n}^*,\ n^{**} \leq \hat{n}^{**};$$

$$n_F^* \leq \hat{n}_F^*,\ n_F^{**} \leq \hat{n}_F^{**};$$

$$n_P^* \leq \hat{n}_P^*,\ n_P^{**} \leq \hat{n}_P^{**}.$$

Where n^* and n^{**} denote the optimal number of firms and socially efficient number of firms under the distribution $F(\cdot)$, while $\hat{n}^*$ and $\hat{n}^{**}$ denote the optimal number of firms and socially efficient number of firms under the distribution $G(\cdot)$, respectively. The subscripts 'F' and 'P' denote the cases of fixed bribe and proportional bribe.

Proof: In the Appendix. ■

The intuition, as mentioned above, is that more dispersed cost distribution implies that firms are becoming more heterogeneous; thus, for a given number of firms, auctions are becoming less competitive than before. As a result, more firms need to be invited to increase competition in an auction.

Our discussion on the information disclosure raises the concern that a corrupt bureaucrat can disguise his corruption through information disclosure, which enables him to manipulate the dispersion of firms' cost distribution. For example, if the *ex ante* distribution of firms' cost is $F(\cdot)$, the optimal number of firms without corruption is n^* and the optimal number of firms with a proportional bribe is n_P^*. Let us consider the case of $n_P^* < n^*$. If the bureaucrat controls information disclosure, then he can manipulate the cost distribution into $G(\cdot)$, which is more dispersed than $F(\cdot)$. In this case, he can request the bribery and choose $\hat{n}_P^*$, which is larger than n_P^* and possibly closer to n^*. That information manipulation can be conducted privately such that the government may still believe that the underlying distribution is $F(\cdot)$, which makes the detection of the bureaucrat's misconduct more difficult.

4.3 Government regulations

The above results show that the effects of corruption on equilibrium competition and social welfare vary across different forms of bribery. They also raise the question of optimal and effective regulations by a central government to ensure sound public services and improve social welfare. In the case of a fixed bribe, although it will not affect the equilibrium competition, it does incur more public spending by the government, which induces social welfare loss. To prevent corruption in that form, the central government needs to conduct a strict audit of the project budget, guarantee sufficient transparency over the entire competition process, and carefully evaluate the claimed costs by firms.

In the case of a proportional bribe, the corrupt bureaucrat favors less or more competition to share higher revenue with the winning firm. The actual impact depends on how much the bureaucrat weights his individual interest. Thus, it may be not efficient for the government to impose some requirements on the minimum number of firms in public procurements. However, if we believe that a corrupt bureaucrat cares more about his personal interest than the government payoff, say $\alpha < 1/2$ in Proposition 3, then he will have an incentive to dampen competition in the auction. In this case, it would be helpful for the central government or legal authorities to impose requirements on the minimum number of firms in public procurement. For example, in the rules and procedures for public procurement in the European Union, a public authority must invite at least five candidates possessing the capabilities required to submit tenders in the restricted procedure.[9]

5 Concluding remarks

The link between corruption and competition is an important issue in public procurements, but it receives insufficient attention in the existing literature. In this paper, we studied the effects of corruption on equilibrium competition and social welfare in a model of a first-price procurement auction. In our model, a bureaucrat runs the auction on behalf of the government, and firms are invited to the auction, which is costly for the government. The bureaucrat may request a bribe from the winning firm, which can either be in the form of a fixed bribe or of a proportional bribe.

First, in the benchmark case of no corruption, whereby the bureaucrat's objective is in line with that of the government, we show the over-invitation result. That is, the

[9] See europa.eu/youreurope/business/public-tenders/rules-procedures/index_en.htm. Updated on Nov. 2015.

optimal number of firms that maximizes the expected government payoff is larger than the efficient number of firms that maximizes social welfare. This result lies in the fact that the government is not modeled as a social planner in our model, and it cares about its own payoff rather than the overall social welfare.

Second, for a corrupt bureaucrat, we show that the effects of corruption on equilibrium competition and social welfare vary across different forms of bribery. Specifically, in the case of a fixed bribe, corruption has no effect on equilibrium competition, as the bribe is fixed and it will not change the bureaucrat's incentive to invite firms. However, it does induce social welfare loss and different resource allocations in equilibrium. This is because, in expectation of the fixed bribe, all firms will mark-up their bids by the same amount of bribe. As a result, it is the government that actually pays the fixed bribe, and higher government spending implies social welfare loss due to the distortion cost of public funds.

In the case of a proportional bribe, corruption will induce either less or more competition in equilibrium. It depends on the weight a bureaucrat gives to his personal interest relative to the government payoff.

The results of our model shed light on public procurement regulations. First, we show that different forms of corruption have markedly different implications on competition and welfare outcomes in procurement auctions. Accordingly, a regulator needs to carefully consider the different regulation rules that target different forms of corruption. Meanwhile, we show that it would be easier for a corrupt bureaucrat to disguise his misconduct if he could manipulate the information released to firms. Therefore, when designing the rules on information disclosure, the regulator needs to conduct a close investigation on the pros and cons of the effects of information disclosure.

Furthermore, this paper also develops a simple and clear framework for analyzing relevant issues in procurement auctions where the number of firms is variable. This approach is different from most work in the existing literature whereby the number of firms is fixed and the focus is on how a bureaucrat manipulates the auction rules in exchange for a bribe. We believe this framework can be readily extended to many other relevant problems, such as corruption in scoring auctions.

Appendix

Proof of Lemma 2:

Proof: It holds that

$$E[X_{1:n}] = \int_0^V x dF_{1:n}(x) = \int_0^V (1 - F(x))^n \, dx,$$

and it implies

$$E[X_{1:n+1} - X_{1:n}] = -\int_0^V F(x)(1 - F(x))^n \, dx < 0.$$

If $f(x) > 0$, then $E[X_{1:n+1} - X_{1:n}]$ is strictly increasing in n. Thus we have $E[X_{1:n}]$ is strictly decreasing and strictly convex in n.

Furthermore, $\lim_{n\to\infty} E[X_{1:n}] = \lim_{n\to\infty} \left[\int_0^V (1 - F(x))^n \, \mathrm{d}x\right] = 0$, as $F(x) \in (0, 1)$ for $x \in (0, V)$, given that $f(x) > 0$. ■

Proof of Lemma 3:

Proof: We first have

$$\begin{aligned} F_{1:n}(x) &= 1 - \bar{F}(x)^n \\ F_{2:n}(x) &= 1 - \bar{F}(x)^{n-1}[1 + (n-1)F(x)] \end{aligned}$$

where $\bar{F}(x) = 1 - F(x)$ is the survival function. It follows that

$$E[X_{2:n} - X_{2:n+1}] = \int_0^V [F_{2:n+1}(x) - F_{2:n}(x)]dx = \int_0^V \frac{F^2(x)}{f(x)} dF_{1:n}(x) > 0.$$

If the distribution $F(\cdot)$ is DRHR, we see that the function $h(x) = \frac{F^2(x)}{f(x)}$ is increasing. Thus, $E[X_{2:n}]$ is strictly decreasing and convex in n. Furthermore, as $\lim_{x\to 0} h(x) = 0$, we have $\lim_{n\to\infty} E[X_{2:n} - X_{2:n+1}] = 0$ from the above lemma. ■

Proof of Lemma 4:

Proof: We have $E[X_{2:n} - X_{1:n}] = \int_0^V [F_{1:n}(x) - F_{2:n}(x)]dx = \int_0^V \frac{F(x)}{f(x)} dF_{1:n}(x)$. Let $h(x) = \frac{F(x)}{f(x)}$, it is increasing under the DRHR assumption, and $\lim_{x\to 0} h(x) = 0$. Following the same proof as in Lemma 3, we obtain the results. ■

Proof of Proposition 1:

Proof: If $F(\cdot)$ is DRHR, $E[R(n)] = E[X_{2:n} - X_{1:n}]$ is strictly decreasing in n, then

$$E[X_{2:n} - X_{1:n}] > E[X_{2:n+1} - X_{1:n+1}] \Leftrightarrow E[X_{2:n} - X_{2:n+1}] > E[X_{1:n} - X_{1:n+1}].$$

From $n^* = \arg\max E[\Pi(n)]$, the optimization condition implies

$$E[\Pi(n^*) - \Pi(n^* - 1)] \geq 0 > E[\Pi(n^* + 1) - \Pi(n^*)],$$

which is equivalent to

$$E[X_{2:n^*-1} - X_{2:n^*}] \geq C(n^*) - C(n^* - 1)$$

and

$$E[X_{2:n^*} - X_{2:n^*+1}] < C(n^* + 1) - C(n^*).$$

Similarly, for $n^{**} = \arg\max E[W(n)]$, we have

$$\frac{1}{1+\lambda} E[X_{1:n^{**}-1} - X_{1:n^{**}}] + \frac{\lambda}{1+\lambda} E[X_{2:n^{**}-1} - X_{2:n^{**}}] \geq C(n^{**}) - C(n^{**} - 1)$$

and

$$\frac{1}{1+\lambda} E[X_{1:n^{**}} - X_{1:n^{**}+1}] + \frac{\lambda}{1+\lambda} E[X_{2:n^{**}} - X_{2:n^{**}+1}] < C(n^{**} + 1) - C(n^{**}).$$

As for all n,

$$E[X_{2:n-1} - X_{2:n}] > \frac{1}{1+\lambda} E[X_{1:n-1} - X_{1:n}] + \frac{\lambda}{1+\lambda} E[X_{2:n-1} - X_{2:n}];$$

thus, the result of $n^* \geq n^{**}$ is obvious. ■

Proof of Lemma 5:

Proof: Denote the new increasing bidding strategy in a symmetric equilibrium by $b_F(x)$, and bidders $i = 2, \cdots, n$ stick to this strategy. Denote $Y = \min\{X_2, \cdots, X_n\} = X_{1:n-1}$, and then $b_F(Y)$ is the lowest bid of those bidders. Bidder 1 wins the auction whenever his bid $\beta < b_F(Y)$ bidder, and he chooses the bid to maximize his expected payoff

$$\mathrm{E}\left[(\beta - x) \cdot \mathbf{1}_{\beta < b_F(Y)}\right] = (\beta - x - B)\left[1 - F_{1:n-1}\left(b_F^{-1}(\beta)\right)\right].$$

Taking the first-order condition with respect to β, it follows that

$$\left[1 - F_{1:n-1}\left(b_F^{-1}(\beta)\right)\right] - (\beta - x - B)\frac{f_{1:n-1}\left(b_F^{-1}(\beta)\right)}{b_F'\left(b_F^{-1}(\beta)\right)} = 0.$$

In symmetric equilibrium, $\beta = b_F(x)$, and thus

$$b'_F(x)\left[1 - F_{1:n-1}(x)\right] - b(x) f_{1:n-1}(x) = -(x+B) f_{1:n-1}(x),$$

or equivalently,

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[b_F(x)\left[1 - F_{1:n-1}(x)\right]\right] = -(x+B) f_{1:n-1}(x).$$

Since $b(V) = V + B$, then

$$b_F(x) = \int_x^V (y+B)\,\mathrm{d}\frac{F_{1:n-1}(y)}{\bar{F}_{1:n-1}(x)} = (x+B) + \int_x^V \frac{\bar{F}_{1:n-1}(y)}{\bar{F}_{1:n-1}(x)}\mathrm{d}y.$$

■

Proof of Lemma 6:

Proof: Denote the new increasing bidding strategy in a symmetric equilibrium by $b_P(x)$, and bidders $i = 2, \cdots, n$ stick to this strategy. Denote $Y = \min\{X_2, \cdots, X_n\} = X_{1:n-1}$, and then $b_P(Y)$ is the lowest bid of those bidders. Bidder 1 wins the auction whenever his bid $\beta < b_P(Y)$ bidder, and he chooses bid to maximize his expected payoff

$$\mathrm{E}\left[\left((1-\eta)\beta - x\right)\cdot \mathbf{1}_{\beta < b_P(Y)}\right] = \left[(1-\eta)\beta - x\right]\left[1 - F_{1:n-1}\left(b_P^{-1}(\beta)\right)\right].$$

Taking the first-order condition with respect to β, it follows that

$$(1-\eta)\left[1 - F_{1:n-1}\left(b_P^{-1}(\beta)\right)\right] - \left[(1-\eta)\beta - x\right]\frac{f_{1:n-1}\left(b_P^{-1}(\beta)\right)}{b'_P\left(b_P^{-1}(\beta)\right)} = 0.$$

In symmetric equilibrium, $\beta = b_P(x)$, and thus

$$b'_P(x)\left[1 - F_{1:n-1}(x)\right] - b_P(x) f_{1:n-1}(x) = -\frac{x f_{1:n-1}(x)}{1-\eta},$$

or equivalently,

$$\frac{\mathrm{d}}{\mathrm{d}x}\left[b_P(x)\left[1 - F_{1:n-1}(x)\right]\right] = -\frac{x f_{1:n-1}(x)}{1-\eta}.$$

Since $b(V) = V/(1-\eta)$, then

$$b_P(x) = \int_x^V \frac{y}{1-\eta}\mathrm{d}\frac{F_{1:n-1}(y)}{\bar{F}_{1:n-1}(x)} = \frac{1}{1-\eta} b(x).$$

■

Proof of Proposition 3:

Proof: If $0 < \eta < \frac{\alpha}{1-\alpha}$, Lemma 7 implies $E[U_P(n)]$ is strictly concave in n. Then, the optimization conditions for n_P^* are

$$\frac{\alpha-(1-\alpha)\eta}{1-\eta}E[X_{2:n-1}-X_{2:n}] \geq \alpha(C(n)-C(n-1))$$

and

$$\frac{\alpha-(1-\alpha)\eta}{1-\eta}E[X_{2:n}-X_{2:n+1}] < \alpha(C(n+1)-C(n)).$$

Compared with the optimization conditions for n^*, we imply that,
If $\frac{\alpha-(1-\alpha)\eta}{1-\eta} \leq \alpha$, i.e., $\alpha \leq 1/2$, then we have $n_P^* \leq n^*$.
If $\frac{\alpha-(1-\alpha)\eta}{1-\eta} > \alpha$, i.e., $\alpha > 1/2$, we have $n_P^* > n^*$. ■

Proof of Proposition 4:

Proof: Apparently, $n_F^{**} = n^{**}$, in which both follow the same optimization conditions

$$\phi(n) \geq C(n) - C(n-1)$$

and

$$\phi(n+1) < C(n+1) - C(n),$$

where $\phi(n) = \frac{1}{1+\lambda}E[X_{1:n-1} - X_{1:n}] + \frac{\lambda}{1+\lambda}E[X_{2:n-1} - X_{2:n}]$. Under proportional bribe, the optimization conditions are

$$\phi(n) + \frac{\alpha\eta}{(1+\alpha)(1-\eta)}E[X_{2:n-1} - X_{2:n}] \geq C(n) - C(n-1)$$

and

$$\phi(n+1) + \frac{\alpha\eta}{(1+\alpha)(1-\eta)}E[X_{2:n} - X_{2:n+1}] < C(n+1) - C(n).$$

Thus, we have $n_P^{**} > n^{**}$. ■

Proof of Lemma 8:

Proof: Let us denote the increasing bidding function by $b_P(x)$ for a firm with cost x. Given other firms that follow this strategy, we derive the equilibrium for firm 1. Denote $Y = \min\{X_2, \cdots, X_n\}$, and then $b_P(Y)$ is the lowest bid of those $n-1$ rival bidders. Bidder 1 wins the auction whenever his bid $\beta < b_P(Y)$. The objective function is

$$E[((1-\eta)b_P(Y) - x) \cdot 1_{\beta < b_P(Y)}] = E[(b_P(Y) - \frac{x}{1-\eta}) \cdot 1_{\beta < b_P(Y)}].$$

It is clear that $\beta = \frac{x}{1-\eta}$ is a weakly dominant strategy. ■

Proof of Proposition 5:

Proof: From Theorem 5.3.1 of Arnold et. al (2008), we have the following recurrence relationship

$$iE[X_{i+1:n}] + (n-i)E[X_{i:n}] = nE[X_{i:n-1}].$$

Therefore,

$$E[X_{2:n-1} - X_{2:n}] = \frac{2}{n}E[X_{3:n} - X_{2:n}] \leq \frac{2}{n}E[Y_{3:n} - Y_{2:n}] = E[Y_{2:n-1} - Y_{2:n}],$$

where the inequality is implied by the result that $E[X_{j:n} - X_{j-1:n}] \leq E[Y_{j:n} - Y_{j-1:n}]$. Thus, we can conclude that $n^* \leq \hat{n}^*$. Similarly, we can also prove the result that $n^{**} \leq \hat{n}^{**}$. From Proposition 2, it is clear that $n_F^* = n^*$ and $\hat{n}_F^* = \hat{n}^*$, and then $n_F^* \leq \hat{n}_F^*$. From Proposition 4, we have $n_F^{**} \leq \hat{n}_F^{**}$.

Now let us prove that $n_P^* \leq \hat{n}_P^*$. First, if n_P^* is the corner solution that $n_P^* = 1$, then $n_P^{**} \leq \hat{n}_P^{**}$ is clearly true as $\hat{n}_P^{**} \geq 1$. Second, if n_P^* is an interior solution such that the optimization conditions for n_P^* are

$$\frac{\alpha - (1-\alpha)\eta}{1-\eta}E[X_{2:n-1} - X_{2:n}] \geq \alpha(C(n) - C(n-1))$$

and

$$\frac{\alpha - (1-\alpha)\eta}{1-\eta}E[X_{2:n} - X_{2:n+1}] < \alpha(C(n+1) - C(n)).$$

Thus, we can conclude that $n_P^* \leq \hat{n}_P^*$. Similar results $n_P^{**} \leq \hat{n}_P^{**}$ can be implied from the optimization conditions as in the Proof of Proposition 4. ■

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